Single Sampling Plans for Attributes

A single sampling plan is a procedure by which a single sample is drawn form a lot and inspected. The lot is accepted if the number of nonconforming units found in the sample is less than or equal to the acceptance number (or a specified limit); otherwise, the lot is rejected. Now let's discuss an approach to find the appropriate single sampling plan.

Scenario

"A company wants to have a single sampling plan that will not accept, more than 10% of the time, material that is 8% defective or worse. In the mean time, this company would like to have at least 95% of chance to accept a submitted lot with 1% or less nonconforming."

Interpretation

AQL = p_1 = 1%, Producer's risk, α = 100% - 95% = 5%

RQL = p_2 = 8%, Consumer's risk, β = 10%

Task

Find sample size, n, and acceptance level, c.

Steps

1. Determine the ratio of p_2/p_1. Here we have the ration calculated as 8.
2. Obtain c, the acceptance level, from the table below by using the nearest ratio of p_2/p_1.

c **acceptance level**	p_1n (Pa=0.95)	p_2n (Pa=0.10)	p_2/p_1
0	0.051	2.30	45.10
1	0.355	3.89	10.96
2	0.818	5.32	6.50
3	1.366	6.68	4.89
4	1.970	7.99	4.06
5	2.613	9.28	3.55
6	3.285	10.53	3.21
7	3.981	11.77	2.96
8	4.695	12.99	2.77
9	5.425	14.21	2.62
10	6.169	15.41	2.50
11	6.924	16.60	2.40
12	7.690	17.78	2.31
13	8.464	18.96	2.24
14	9.246	20.13	2.18
15	10.04	21.29	2.12

Table 1. Table for a two-point design of a single sampling plan with α at 0.05 and β at 0.10*

* Source: From F. E. Grubbs, "On designing single sampling inspection plans," Annals of Mathematical Statistics 20, 1949, p.256

Here, with a ratio of p_1/p_2 equal to 8, we can choose to have c either at 1 or 2.

3. Determine the sample size for each c value by holding either α or β at specified level.

   If let c = 1 and hold β at 0.10, the sample size can be calculated as:

   $$n = (p_2 n) / p_2 = 3.89/0.08 \approx 49.$$

   If use the same c value but hold α at 0.05, the sample size can be calculated as:

   $$n = (p_1 n) / p_1 = 3.55/0.01 \approx 36.$$

   If let c = 2 and hold β at 0.10, the sample size can be calculated as:

   $$n = (p_2 n) / p_2 = 5.32/0.08 \approx 67.$$

   If use the same c value but hold α at 0.05, the sample size can be calculated as:

   $$n = (p_1 n) / p_1 = 0.818/0.01 \approx 82.$$

4. State the eligible sampling plans and choose the one fit most closely to the specification. According to the above, there are four eligible sampling plans with various sample sizes and acceptance levels, they are:

   plan A- n = 49, c = 1,
   plan B- n = 36, c = 1,
   plan C- n = 67, c = 2, and
   plan D- n = 83, c = 2.

   It should be noted that plan A and C were obtained by holding their consumer's risk at 10% and plan B and D were obtained by holding their producer's risk at 5%. In order to find the most appropriate sampling plan, we need to calculate the producer's risks for plan A and C, and consumer's risks for B and D. Using plan B as an example, its consumer's risk can be calculated as:

$$\beta = \Pr(x \le 1) = \sum_{x=0}^{1} \binom{36}{x} 0.08^x (1-0.08)^{36-x} = 0.21.$$

Result

The four eligible sampling plans and their respective producer's risks and consumer's risks is given in the table below.

Plan	Sample size	Acceptance level	Producer's risk	Consumer's risk
A	49	1	0.09	0.10
B	36	1	0.05	0.21
C	67	2	0.03	0.10
D	82	2	0.05	0.04

Table 2. Eligible sampling plans and associated risks.

Since the company would like to have $\alpha \leq 5\%$ and $\beta \leq 10\%$ with respect to the associated quality levels, they can choose to have either plan C or plan D. The OC curve associated with plan C is given below in Figure 5.

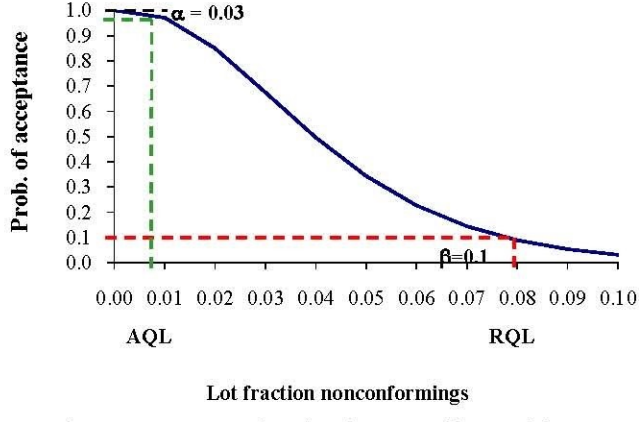


Figure 5. The OC curve of a single sampling with n = 67 and c =2